

# On the Phase Diagram of Massive Yang-Mills

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**Abstract:** The phases of a lattice gauge model for the massive Yang-Mills are investigated. The phase diagram supports the recent conjecture on the large energy behavior of nonlinearly realized massive gauge theories (i.e. mass *à la* Stückelberg, no Higgs mechanism), envisaging a Phase Transition (PT) to an asymptotically free massless Yang-Mills theory.

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# 1 Introduction

A novel approach to the massive Yang-Mills gauge theory has been proposed [1], where the divergences are consistently removed in the loop expansion. The removal strategy follows close the method used recently for the nonlinear sigma model [2]. It consists in the subtraction of the *pure* pole parts of the properly normalized one-particle-irreducible amplitudes regularized in  $D$  dimensions. The mathematical tools are standard, but the proof that the technique is consistent is rather involved [2], [3], [4].

Although the subtraction method is consistent with the Slavnov Taylor Identities, locality and a new, *ad hoc* derived, Local Functional Equation, the perturbative series seems to be inadequate for high energy processes, thus casting some doubts on the validity of unitarity (although  $SS^\dagger = 1$  order by order in the loop expansion). It has been recently suggested [5] that the cause of this is due to some singularities (phase transitions) present in the parameters space ( $\beta := \frac{4}{g^2}, m^2$ ). According to this scenario one can approach the theory with the usual perturbative loop expansion for low-energy processes, while the high energy processes are described by the massless Yang-Mills theory with no remnants of the longitudinal polarizations. The transition between the two regimes may be studied by the lattice simulation. This is attempted in the present paper, where we try to study the model in the non perturbative regime.

The present work was motivated by the arguments just outlined, but its contents and results are valid independently from them. In fact it opens new perspectives on the lattice gauge theories.

An intensively studied lattice gauge model [6]- [13] turns out to be the perfect tool for the simulation of the massive Yang-Mills (i.e. mass *à la* Stückelberg). We confirm the existence of a Transition Line (TL) which separates a *confined* phase from one with physical vector boson states. The phase LT has an end-point around ( $\beta \sim 2.2, m^2 \sim 0.381$ ): for smaller  $\beta$  there is a smooth transition (crossover) from one phase to the other, while for larger  $\beta$  there are numerical indications of singularities in the derivatives with respect of  $m^2$  and of  $\beta$  of the energy and of the order parameter (the  $m^2$  derivative of the free energy). The deconfined phase is studied by using the correlators of *gauge invariant* fields. This allows a full gauge invariant approach to the model. We give numerical evidence of the existence of iso-vector modes for the spin one (no spin zero is present). For

the isoscalar fields there is a faint, but persistent, signal of an energy gap both for spin one and zero. Far from the LT these excitations in the isoscalar channels are compatible with the threshold of two iso-vector spin one modes. However near the TL the energy gap in the isoscalar channels is lower than the threshold, thus suggesting the existence of both spin one and spin zero bound states. This effect happens in a band attached to the TL: for large  $\beta$  (i.e. higher than the end point value) the band is very narrow, while in the crossover region (low  $\beta$ ) the onset of bound states is smooth and on a wider region. The tantalizing question is whether this interesting region of the phase space will ever be reached by experiments and the presence of bound states confirmed.

## 2 The Lattice Model

The field theory (for the  $SU(2)$  group) in the continuum is [1]

$$S_{YM} = \frac{1}{g^2} \int d^4x \left( -\frac{1}{4} G_{a\mu\nu}[A] G_a^{\mu\nu}[A] + \frac{M^2}{2} (A_{a\mu} - F_{a\mu})^2 \right), \quad (1)$$

where in terms of the Pauli matrices  $\tau_a$

$$A_\mu = \frac{\tau_a}{2} A_{a\mu}, \quad F_\mu = \frac{\tau_a}{2} F_{a\mu} := i\Omega \partial_\mu \Omega^\dagger. \quad (2)$$

$\Omega(x)$  is an element of the  $SU(2)$  group, parameterized by four real fields

$$\Omega = \phi_0 + i\tau_a \phi_a, \quad \implies \phi_0^2 + \vec{\phi}^2 = 1. \quad (3)$$

We have

$$F_{a\mu} = 2(\phi_0 \partial_\mu \phi_a - \partial_\mu \phi_0 \phi_a + \epsilon_{abc} \partial_\mu \phi_b \phi_c). \quad (4)$$

The action in eq.(1) is invariant under  $g_L(x) \in SU(2)_L$  local-left and  $g_R \in SU(2)_R$  global-right transformations

$$SU(2)_L \left\{ \begin{array}{l} \Omega'(x) = g_L(x) \Omega(x) \\ A'_\mu(x) = g_L(x) A_\mu g_L^\dagger(x) \\ \quad + i g_L(x) \partial_\mu g_L^\dagger(x) \end{array} \right., \quad SU(2)_R \left\{ \begin{array}{l} \Omega'(x) = \Omega(x) g_R^\dagger \\ A'_\mu(x) = A_\mu(x) \end{array} \right. . \quad (5)$$

The theory is not renormalizable due to the non-polynomial dependence on  $\vec{\phi}$  explicit in eq. (4). We refer to the Refs. [1] and [2] on the new strategy suggested for the consistent subtraction of all the ultraviolet divergences.

The lattice model is constructed by assuming a nearest neighbor interaction and by requiring a naïve mapping into the action (1) in the limit of zero lattice spacing. It is very important to construct a model invariant under the discretized versions of eqs. (5). The link variable is taken to be

$$U(x, \mu) \simeq \exp(-iaA_\mu(x)). \quad (6)$$

Both  $U(x, \mu)$  and the site variable  $\Omega(x)$  are elements of  $SU(2)$ . Thus the action is ( $\beta = \frac{4}{g^2}$ )

$$S_E = \frac{\beta}{2} \Re e \sum_{\square} Tr(1 - U_{\square}) + \frac{\beta}{2} M^2 a^2 \Re e \sum_{x\mu} Tr \left\{ 1 - \Omega(x)^\dagger U(x, \mu) \Omega(x + \mu) \right\}, \quad (7)$$

where the sum over the plaquette is the Wilson action [14] and the mass term has the (Euclidean) continuum limit

$$\begin{aligned} & \frac{\beta}{2} M^2 a^2 \Re e \sum_{x\mu} Tr \left\{ 1 - \Omega(x)^\dagger U(x, \mu) \Omega(x + \mu) \right\} \\ & \rightarrow \frac{M^2}{g^2} \int d^4x Tr \left\{ (A_\mu - i\Omega \partial_\mu \Omega^\dagger)^2 \right\}. \end{aligned} \quad (8)$$

In the simulation the  $Tr\{1\}$  is omitted. Thus the action becomes

$$S_E \rightarrow -\frac{\beta}{2} \Re e \sum_{\square} Tr(U_{\square}) - \frac{\beta}{2} m^2 \Re e \sum_{x\mu} Tr \left\{ \Omega(x)^\dagger U(x, \mu) \Omega(x + \mu) \right\} \quad (9)$$

From now on the dimensionless parameters are  $\beta$  and  $m^2$ . We work in  $D = 4$ , however the symbol  $D$  is kept in some equations. In the paper we will consider also the model with  $m^2 \rightarrow -m^2$ .

### 3 Simulation

The partition function is obtained by summing over all configurations given by the link variables and the gauge field  $\Omega$

$$Z[\beta, m^2, N] = \sum_{\{U, \Omega\}} e^{-S_E}, \quad (10)$$

where  $N$  is the number of sites.

In principle the integration over  $\Omega(x)$  is redundant, since by a change of variables ( $U_\Omega(x, \mu) := \Omega(x)^\dagger U(x, \mu) \Omega(x + \mu)$ ) we can factor out the volume of the group.  $Z[\beta, m^2, N]$  becomes

$$\left[ \sum_{\{\Omega\}} \right] \sum_{\{U\}} \exp \beta \left( \frac{1}{2} \Re e \sum_{\square} Tr\{U_{\square}\} + \frac{1}{2} m^2 \Re e \sum_{x\mu} Tr\{U(x, \mu)\} \right). \quad (11)$$

In eq. (11) the integration over  $\Omega$  has disappeared; consequently  $\Omega$  in eq. (10) does not describe any degree of freedom. In that respect we are at variance with other approaches to the same action (7) as in [6]- [13], where the field  $\Omega$  is thought of as a Higgs field with frozen length. In eq. (10) we force the integration over the gauge orbit  $U_\Omega$  by means of the explicit sum over  $\Omega$ . In doing this we gain an interesting theoretical setup of the model; in practice, our formalism is fully gauge invariant (Section 5). Moreover by forcing the integration over the gauge orbit  $U_\Omega$  we get results which are less noisy than those obtained by using only the integration over the link variables in (11).

## 4 Functionals and Order Parameter

In this model we can study the energy-per-site functional

$$\begin{aligned} E &= \frac{1}{N} \frac{\partial}{\partial \beta} \ln Z \\ &= \frac{1}{2N} \left\langle \Re e \sum_{\square} Tr\{U_{\square}\} + m^2 \sum_{x\mu} Tr\{\Omega^\dagger(x)U(x,\mu)\Omega(x+\mu)\} \right\rangle, \end{aligned} \quad (12)$$

where  $\langle \rangle$  denotes the mean value by the Boltzmann weight of eq. (10). Moreover we introduce the *magnetization*, i.e. the response to the applied  $m^2$

$$\mathfrak{C} = \frac{1}{DN\beta} \frac{\partial}{\partial m^2} \ln Z = \frac{1}{2ND} \left\langle \Re e \sum_{x\mu} Tr\{\Omega^\dagger(x)U(x,\mu)\Omega(x+\mu)\} \right\rangle. \quad (13)$$

Then we have the plaquette energy

$$E_P = \frac{2}{D(D-1)N} \left\langle \frac{1}{2} \Re e \sum_{\square} Tr\{U_{\square}\} \right\rangle = \frac{2}{D(D-1)} [E - Dm^2\mathfrak{C}]. \quad (14)$$

There are some simple properties that will be of some help in the sequel. Under the mapping

$$U(x,\mu) \rightarrow -U(x,\mu) \quad (15)$$

the Wilson action is invariant while the mass part changes sign. The measure of the group integration is invariant, then we have from eqs. (10), (12) and (13)

$$\begin{aligned} Z[\beta, -m^2, N] &= Z[\beta, m^2, N] \\ E[\beta, -m^2, N] &= E[\beta, m^2, N] \\ \mathfrak{C}[\beta, -m^2, N] &= -\mathfrak{C}[\beta, m^2, N]. \end{aligned} \quad (16)$$

We argue that  $\mathfrak{C}$  is a valid order parameter. This will be shown in the next Sections. We briefly recollect the main points. Its susceptibility  $(\partial\mathfrak{C}/\partial m^2)$  has a cusp-like behavior in  $m^2$  on the TL for  $\beta$  greater than the end point value. It is odd under the change of sign of  $m^2$ . Moreover the implementation of the global  $SU(2)_R$  symmetry is drastically different in the far away regions where  $\mathfrak{C} \sim 0$  and  $\mathfrak{C} \pm 1$ . In the first region  $SU(2)_R$  charged fields are screened or confined, while in the second deconfined modes are present.

## 5 The Vector Meson Fields

Our approach allows the presence of  $SU(2)_L$  gauge invariant fields. Let us consider

$$C(x, \mu) := \Omega^\dagger(x)U(x, \mu)\Omega(x + \mu) = C_0(x, \mu) + i\tau_a C_a(x, \mu), \quad (17)$$

which, according to eqs.(5), are invariant under local  $SU(2)_L$  transformations.  $C_0(x, \mu)$  is the mass term density in the action (9) and it is a  $SU(2)_R$ -scalar (isoscalar), while  $C_a(x, \mu)$  are vectors under the same group of transformations (isovectors). Since  $C(x, \mu) \in SU(2)$ , we get that all fields are real and constrained by

$$C_0(x, \mu)^2 + \sum_a C_a(x, \mu)^2 = 1. \quad (18)$$

Moreover we expect the vacuum to be invariant under  $SU(2)_R$  global transformations (5) and therefore

$$\langle C_a(x, \mu) \rangle = 0, \quad a = 1, 2, 3, \quad \forall(x, \mu). \quad (19)$$

The order parameter (13)

$$\mathfrak{C} = \frac{1}{DN} \sum_{x\mu} \langle C_0(x, \mu) \rangle \quad (20)$$

is the conjugate of the mass parameter  $m^2$ . According to eq. (16) we expect that

$$\lim_{m^2=0} \mathfrak{C} = 0, \quad \lim_{m^2 \rightarrow \infty} \mathfrak{C} = 1. \quad (21)$$

Beside the order parameter, it is important to study the following connected correlators. They will provide the essential characterization of the deconfined phase of the system.

$$C_{ab,\mu\nu}(x, y) := \langle C_a(x, \mu) C_b(y, \nu) \rangle_C$$

$$\begin{aligned}
C_{0b,\mu\nu}(x, y) &:= \left\langle C_0(x, \mu) C_b(y, \nu) \right\rangle_C \\
C_{00,\mu\nu}(x, y) &:= \left\langle C_0(x, \mu) C_0(y, \nu) \right\rangle_C.
\end{aligned} \tag{22}$$

In order to investigate the transition between phases we consider also the susceptibility

$$\begin{aligned}
\frac{\partial}{\partial m^2} \mathfrak{C} &= \frac{\beta}{DN} \sum_{x\mu} \sum_{y\nu} \left\langle C_0(x, \mu) C_0(y, \nu) \right\rangle_C \\
&= \frac{\beta}{DN} \sum_{x\mu} \sum_{y\nu} \left( \left\langle C_0(x, \mu) C_0(y, \nu) \right\rangle - \left\langle C_0(x, \mu) \right\rangle \left\langle C_0(y, \nu) \right\rangle \right). \tag{23}
\end{aligned}$$

It should be noticed that the mean square error of  $\mathfrak{C}$  is related to its derivative

$$\frac{\partial}{\partial m^2} \mathfrak{C} = \beta DN \left\langle \left( \mathfrak{C} - \langle \mathfrak{C} \rangle \right)^2 \right\rangle. \tag{24}$$

This relation is very important for numerical simulations. If the derivative of  $\mathfrak{C}$  had a finite limit for  $N \rightarrow \infty$ , the standard deviation would have a  $1/\sqrt{N}$  behavior. If instead the derivative diverges then the standard error might not have a decreasing behavior by increasing the lattice size  $N$ . If this is the case, then the calculation of the derivative by using the heat bath yields a noisy signal. The noise might not decrease by increasing the lattice size, as expected in the normal case.

### 5.1 The $SU(2)$ Right Symmetry

If the  $SU(2)_R$  symmetry is unitarily implemented then we expect

$$\begin{aligned}
C_{ab,\mu\nu}(x, y) &= 0, \quad \text{if } a \neq b, \quad a, b = 1, 2, 3 \\
C_{0b,\mu\nu}(x, y) &= 0.
\end{aligned} \tag{25}$$

The energy gap in the correlator in  $C_{00,\mu\nu}(x, y)$  might set on above the two-particle threshold. However there is an interesting possibility that the gap (both for spin one and spin zero) shows up below this threshold, thus suggesting the existence of bound states.

### 5.2 The Continuum Limit of $C$

By a similar argument used in Section 2 we study the continuum limit of  $C(x, \mu)$ . We have

$$C(x, \mu) = \Omega^\dagger(x) (1 - ia A_\mu(x)) (\Omega(x) + a \partial_\mu \Omega) + \mathcal{O}(a^2). \tag{26}$$

Thus for  $C_1, C_2, C_3$  one gets

$$i\tau_a C_a(x, \mu) = -ia\Omega^\dagger \left( A_\mu(x) - i\Omega \partial_\mu \Omega^\dagger \right) \Omega + \mathcal{O}(a^2). \quad (27)$$

While for  $C_0$  we can use the result of Section 2, eqs. (7) and (8)

$$C_0(x, \mu) = 1 - \frac{a^2}{4} \text{Tr} \left\{ (A_\mu - i\Omega \partial_\mu \Omega^\dagger)^2 \right\} + \mathcal{O}(a^4). \quad (28)$$

Notice that the dominant terms in eqs. (27) and (28) are  $SU(2)$  local-left-invariant.

The continuum limit of  $C_0(x, \mu)$  and the expression of the order parameter in eq. (13) suggests that in the region  $\mathcal{C} \sim 0$  where confinement (or screening) occurs there is condensation of vector meson pairs, while in the deconfined region, where  $\mathcal{C} \sim 1$ , vector mesons do not condense and are realized as particles.

## 6 Note on Symmetry Breaking

The symmetry of the model and of the partition function is rather interesting. The  $SU(2)_L$  left transformations (5) correspond to the local symmetries of the action, while the  $SU(2)_R$  transformations (5) can only be global symmetries, due to the fact that the mass term in  $S_E$  (eq. (7)) breaks the local  $SU(2)_R$  symmetry. For decreasing mass parameter  $m^2$  we expect the onset of a local  $SU(2)_R$  symmetry. Then the fields  $C(x, \mu)$ , by construction (17), transform according to a  $SU(2) \otimes SU(2) \sim O(4)$  group of transformations

$$C(x, \mu)' = g_R(x) C(x, \mu) g_R(x + \mu)^\dagger. \quad (29)$$

This fact has far reaching consequences. That is, in the limit of zero mass, only closed loops of  $C(x, \mu)$  fields have non zero expectation value [15]. In particular all the correlators in eqs. (22) are zero unless  $y = x + \mu$  and  $y + \nu = x$ , i.e.  $\nu = -\mu$  and  $C(y, \nu) = C(x, \mu)^\dagger$ . But then the  $O(4)$  on the set  $\{C_0(x), C_a(x)\}$  imposes

$$C_{00, \mu\nu}(x, y) \simeq C_{11, \mu\nu}(x, y) = C_{22, \mu\nu}(x, y) = C_{33, \mu\nu}(x, y). \quad (30)$$

The numerical simulations show that the onset of a local  $SU(2)_R$  is very rapid when one crosses the line of PT. It becomes smooth for small  $\beta$ , after the end point.

When the mass parameter becomes large the  $O(4)$  symmetry will be lost and only the global  $SU(2)_R$  invariance will survive and therefore  $C_{00, \mu\nu}(x, y)$  will be substantially different from the  $SU(2)_R$  - vector components.



## 7 Survey

We have performed standard Monte Carlo simulations for the model based on the action (9). Heat bath has been used for the updating. Normally we have saved a configuration every fifteen updatings for a total of 10,000 measures.

We considered cubic 4-dimensional lattices with periodic boundary conditions of different sizes:  $4^4, 6^4, 8^4, 12^4, 16^4, 24^4$ . We have chosen the size on the basis of the precision needed. Typically a  $6^4$  lattice size provides sensible results if  $(\beta, m^2)$  is far away from the TL.

We have built a bird's-eye view of the region  $\beta \in [1, 4], m^2 \in [0, 8]$  of some global quantities of the system. The quantities studied are those described in Section 4, i.e. the energy per site  $E$  (12), the order parameter  $\mathfrak{C}$  (13) and their derivatives with respect to  $m^2$  and  $\beta$  as in eq. (23)

$$\begin{aligned}
\frac{\partial}{\partial m^2} \mathfrak{C} &= \frac{\beta}{D} \left\langle \left( \frac{1}{\sqrt{N}} \sum_{x\mu} C_0(x, \mu) - \left\langle \frac{1}{\sqrt{N}} \sum_{x\mu} C_0(x, \mu) \right\rangle \right)^2 \right\rangle \\
\frac{\partial}{\partial m^2} E &= D(1 + \beta \frac{\partial}{\partial \beta}) \mathfrak{C} \\
&= D \mathfrak{C} \\
&\quad - \beta \left\langle \left[ \frac{1}{\beta \sqrt{N}} S_E - \left\langle \frac{1}{\beta \sqrt{N}} S_E \right\rangle \right] \left[ \frac{1}{\sqrt{N}} \sum_{x\mu} C_0(x, \mu) - \left\langle \frac{1}{\sqrt{N}} \sum_{x\mu} C_0(x, \mu) \right\rangle \right] \right\rangle \\
\frac{\partial}{\partial \beta} E &= \left\langle \left[ \frac{1}{\beta \sqrt{N}} S_E - \left\langle \frac{1}{\beta \sqrt{N}} S_E \right\rangle \right]^2 \right\rangle.
\end{aligned} \tag{31}$$

Notice that

$$\begin{aligned}
\frac{\partial}{\partial \beta} \mathfrak{C} &= \frac{1}{ND} \frac{\partial}{\partial \beta} \left\langle \left( \sum_{x\mu} C_0(x, \mu) \right) \right\rangle = -\frac{1}{D} \\
&\quad \left\langle \left( \frac{1}{\sqrt{N}} \sum_{x\mu} C_0(x, \mu) - \left\langle \frac{1}{\sqrt{N}} \sum_{x\mu} C_0(x, \mu) \right\rangle \right) \left( \frac{1}{\beta \sqrt{N}} S_E - \left\langle \frac{1}{\beta \sqrt{N}} S_E \right\rangle \right) \right\rangle.
\end{aligned} \tag{32}$$

A survey on the parameters space shows a clear phase change across the line represented in Fig. 1. In particular the line represents the *loci* where the dependence of the order parameter  $\mathfrak{C}$  from  $m^2$  has a marked inflection. The line is stable under the change of the size, for instance from  $6^4$  through  $24^4$ . A throughout study has shown that both energy and order parameter are continuous everywhere including on the TL. Fig. 2 describes the dependence

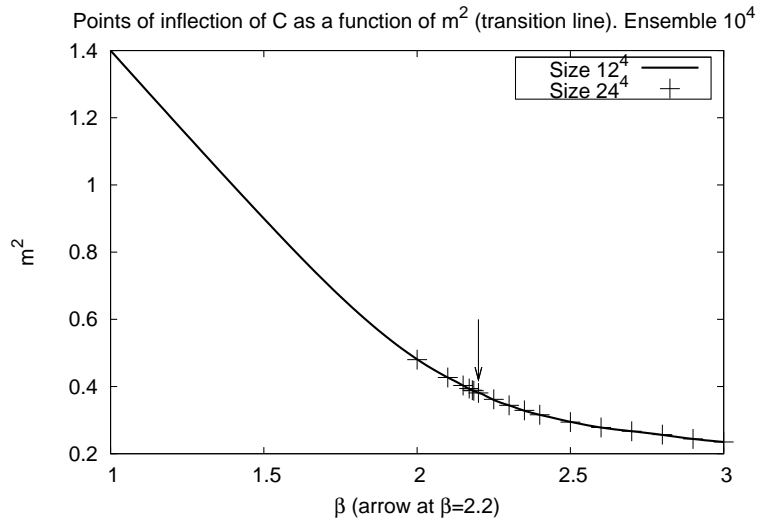


Figure 1: Phase diagram

on  $m^2$  of  $\mathfrak{C}$  and its derivative. All the first derivatives have a cusp behavior for  $\beta \geq 2.2$  while they are smooth for  $\beta < 2.2$ . Fig. 3 exemplifies the situation for  $\beta = 1.5$  and  $\beta = 3$ . There is some evidence for an end point at  $\beta \sim 2.2, m^2 \sim 0.381$ , linked to the crossover point evidenced by early works on SU(2)-QCD simulations [16].

## 8 Numerical Results

The numerical analysis of Sect. 7 confirms the results obtained in previous works about the TL, with some minor discrepancies, as the position of the end point. On the *vexata quaestio*, concerning the order of the PT across the line and for  $\beta \geq 2.2$ , our numerical evidence is not very conclusive, although we would be more in favor of a second order type. The present Section is devoted to the new and surprising results. They show that the model is indeed a faithful simulation of the massive Yang-Mills gauge theory and moreover that unexpected and non trivial features can be obtained in a region of the phase diagram inaccessible by perturbation theory.

We study the operators (zero three-momentum)

$$C_{a,\mu}(t) := \frac{1}{\sqrt{N^{\frac{3}{4}}}} \sum_{\vec{x}} C_a(\vec{x}, x_4, \mu) \Big|_{x_4=t}, \quad a = 0, 1, 2, 3, \quad \mu, \nu = 1, 2, 3, 4. \quad (33)$$

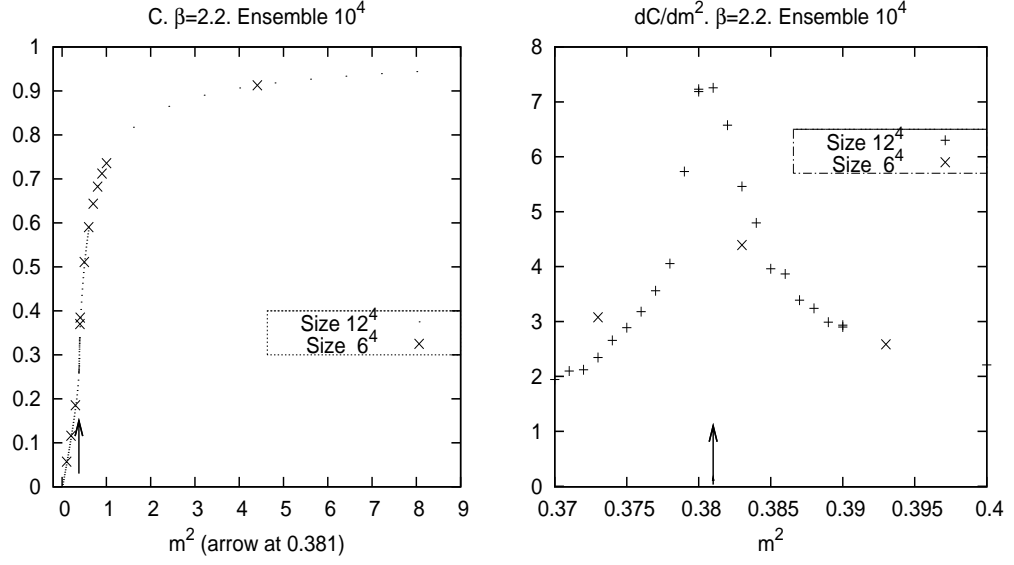


Figure 2:  $\mathfrak{C}$  and  $\frac{\partial \mathfrak{C}}{\partial m^2}$  at  $\beta = 2.2$  for size  $6^4$  and  $12^4$ .

In particular we consider the two-point correlators

$$C_{ab,\mu\nu}(t) := \left\langle C_{a,\mu}(t+t_0) C_{b,\nu}(t_0) \right\rangle_C. \quad (34)$$

Numerical simulations support the selection rules

$$\begin{aligned} C_{0b,\mu\nu}(t) &= 0 \\ C_{ab,\mu\nu}(t) \Big|_{a \neq b} &= 0, \quad a, b = 1, 2, 3 \end{aligned} \quad (35)$$

imposed by the global  $SU(2)_R$  invariance. Moreover, according to eqs. (18), (21) and (30) we get the limit values

$$\lim_{m^2=0} C_{aa,\mu\nu}(0) = 0.25\delta_{\mu\nu}, \quad a = 0, 1, 2, 3. \quad (36)$$

while for  $t > 0$  in the same limit the two-point functions are expected to vanish according to the discussion of Section 6.

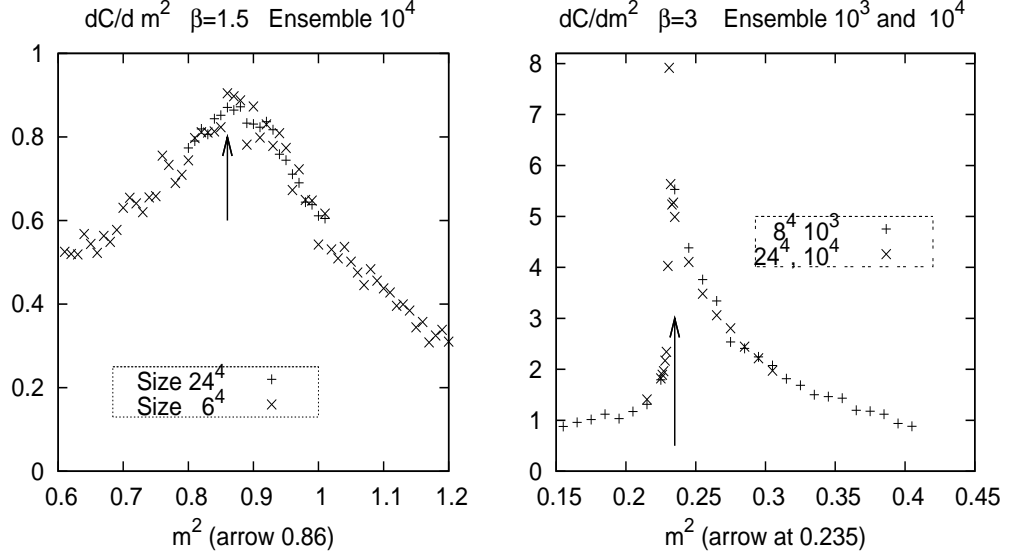


Figure 3:  $\frac{\partial \mathcal{C}}{\partial m^2}$  at  $\beta = 1.5$  and  $\beta = 3$  for different lattice sizes.

The spin analysis is done by decomposing the correlators into a spin one and spin zero parts (dots stand for pairs of iso-indexes 00 or 11,22,33 )

$$C_{\dots,\mu\nu}(t) = V_{\dots}(t)(\delta_{\mu\nu} - \delta_{4\mu}\delta_{4\nu}) + S_{\dots}(t)\delta_{4\mu}\delta_{4\nu}. \quad (37)$$

We fit the amplitudes by a single exponential form

$$F(t) = a + be^{-t\Delta}. \quad (38)$$

A more complex analysis is not at reach with the data at hand. However the form turns out to be sufficient for most of the cases that have been considered. The energy gap  $\Delta$  is obtained from a fit on the correlator (34) evaluated on  $10^4$  configurations. Typical results are shown in Figs. 4 and 5.

The energy gaps have been evaluated for several values of  $(\beta, m^2)$ . Figs. 6 and 7 represent the energy gaps as function of  $m^2$ . Several features are

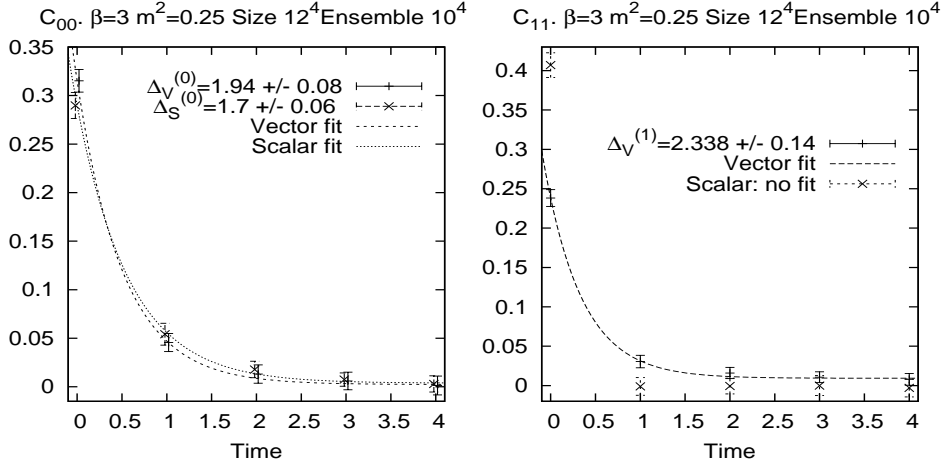


Figure 4: Correlators at  $\beta = 3$  and  $m^2 = 0.25$ :  $C_{00}$  for the isoscalar part and  $C_{11}$  for the isovector according to eq. (37). TL is at  $m^2 = 0.235$ . The numerical values are the energy gaps for spin 1 ( $\Delta_V^{(0)}, \Delta_V^{(1)}$ ) and spin 0 ( $\Delta_S^{(0)}$ ).

present in all the cases we have considered. i) In the deconfined region and far from the TL the isovector correlator is due to a spin one mode with energy gap  $\Delta \simeq |m|$ , while the isoscalar correlator has both spin one and spin zero energy gaps, consistent with a two-vector-meson threshold. ii) Near the TL the isoscalar gaps become smaller than the threshold, thus suggesting the presence of bound states. iii) Across the TL there is a rapid increase of the gaps: within the errors all correlators vanish for  $t > 0$  and a  $O(4)$  symmetry is restored. The change of phase is much more rapid for  $\beta = 3$  than for  $\beta = 1.5$  (the change of scale of  $m^2$  in Figs. 6 and 7 should be properly accounted for).

## 9 Conclusions

We have investigated the deconfined phase of a massive Yang-Mills model by using a set of gauge invariant fields. We give evidence of a TL in the pa-

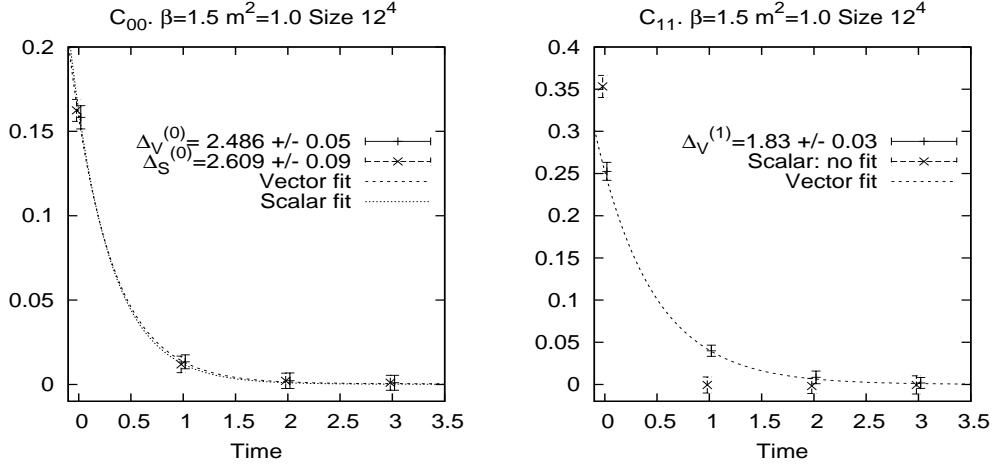


Figure 5: Correlators at  $\beta = 1.5$  and  $m^2 = 1.0$  (TL is at  $m^2 = 0.85$ ). See caption of Fig.4.

rameters space  $(\beta, m^2)$ . An order parameter  $\mathcal{C}$  is introduced (the response to the  $m^2$  parameter) which is  $\sim 1$  in the deconfined region (large  $m^2$ ), while  $\sim 0$  below the TL (small  $m^2$ ). The vanishing of the order parameter corresponds to the condensation of pairs of vector mesons. Far from this line the spectrum consists of an isovector spin one meson and of two-particle states in the isoscalar spin one and spin zero channels. Moreover there is some evidence of bound states near the TL in the isoscalar channels for both spin states. The presence of a discontinuity line confirms the conjecture on the existence of two regimes: a low energy where the loop expansion is valid and an extreme region where massless Yang-Mills works. The last point is a very important step forward for the understanding of high energy processes.

## 10 Acknowledgements

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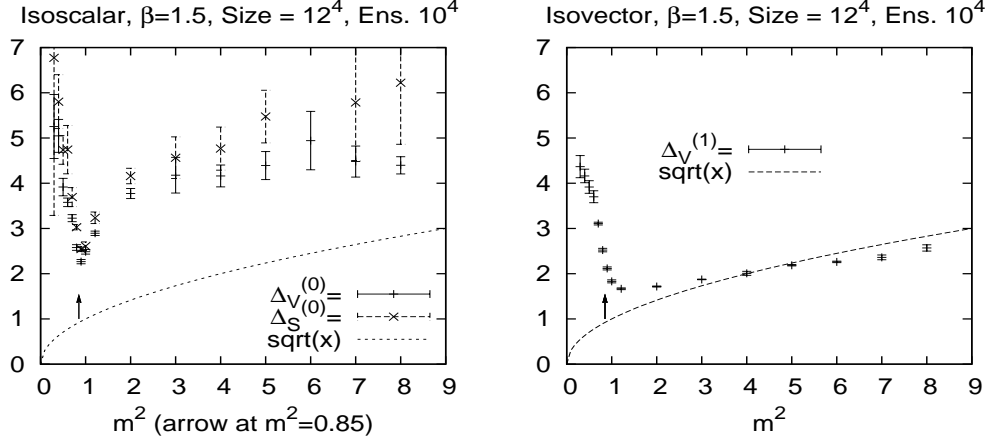


Figure 6: On the y-axis the energy gaps of isoscalar spin one and spin zero (left) and isovector spin one (right) as function of  $m^2$  for fixed  $\beta = 1.5$  (TL at  $m^2 = 0.85$ ). The energy gaps have been extracted from fits as in Figs. 4 and 5 by using the expression in eq. (38). The line is  $\sqrt{|m^2|}$  and it is very close to the spin 1 isovector energy gap in the region away from the phase transition.

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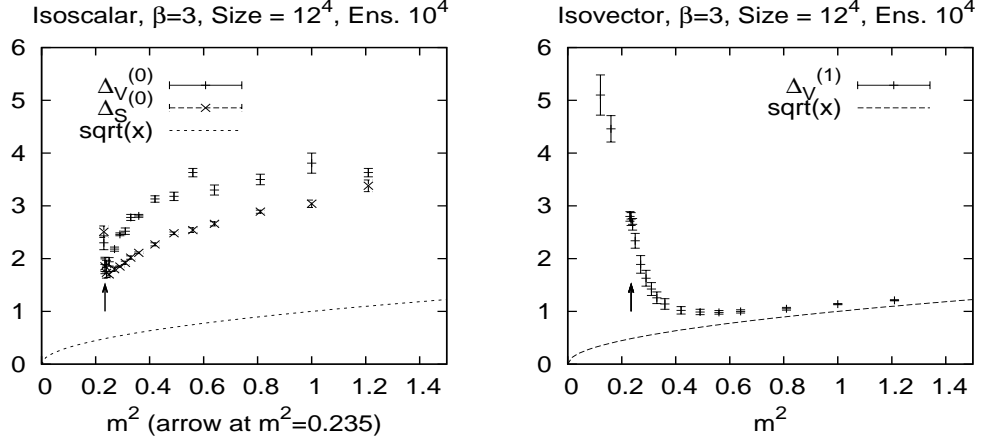


Figure 7: Energy gaps of isoscalar spin one and spin zero (left) and isovector spin one (right) as function of  $m^2$  for fixed  $\beta = 3$  (TL at  $m^2 = 0.235$ ).

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